Linear Approximations to the Quadratic Almost Ideal Demand System: Results from Japanese Demand for Food Including Rice

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Abstract

This paper investigates linear approximations to the recently popular quadratic almost ideal demand system (QUAIDS) by proposing a new composite variable and conducting a simulation study based on Japanese demand for food including rice. The linear approximations are especially useful when one uses nonstationary time series, to which nonlinear systems are difficult to apply properly. The new composite variable performs well in combination with the price indices appropriate for linearizing the almost ideal demand system. The QUAIDS can be linearly approximated on a practical basis if the appropriate combinations of composite variables and elasticity formulas are employed and the base point is set to the point of evaluation.

JEL Classification: C51; D12

Key Words: Quadratic almost ideal demand system, Linear approximation, Composite variable, Monte Carlo experiment

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Introduction

The quadratic almost ideal demand system (QUAIDS), which was derived from utility maximization by Banks et al. (1997), not only retains the desirable properties of the popular almost ideal demand system (AIDS) of Deaton and Muellbauer (1980) nested within it but also has the additional advantage of being potentially more versatile in modeling consumer expenditure patterns. The QUAIDS shares with the AIDS the properties of exact nonlinear aggregation across consumers and second-order flexibility (Diewert, 1974). In addition, quadratic in the logarithm of total expenditure, the QUAIDS allows such situations where the increase in expenditure would change a luxury to a necessity. In the AIDS, on the other hand, expenditure elasticities are independent of expenditure level.

The useful properties of the QUAIDS may account for the fact that it has recently been used in a substantial number of studies. For example, its application to (pseudo-) panels or repeated cross sections was provided by Lewbel (1995), Banks et al. (1996), Blundell and Robin (1999, 2000), Denton et al. (1999), Moro and Sckokai (2000), Luo (2002), Lyssiotou et al. (2002), Tiezzi (2002), Lyssiotou (2003), Nicol (2003), Unayama (2004), and Dhar and Foltz (2005), its application to cross sections by Michelini (1999, 2001), Cranfield et al. (2003), and Abdulai and Aubert (2004), and its application to time series by Jones and Mazzi (1996), Fisher et al. (2001), and Karagiannis and Velentzas (2004). In most of these applications, the quadratic terms of the system are found important in describing consumer behavior.

The empirical literature shows that the AIDS has been approximated at the estimation stage to be linear in parameters in many applications including Deaton and Muellbauer (1980), Blanciforti and Green (1983), Attfield (1985), Moschini and Meilke (1989), Chambers (1990), Nicol (1991), Nichèle and Robin (1995), Cheshire and Sheppard (2002), Moosa and Baxter (2002), Capps et al. (2003), Deschamps (2003), Irwin (2003), and West and Williams (2004). Along with other desirable properties, as Moschini (1995) remarked, the practice of linear estimation is a major reason for the continued popularity of the AIDS.

With current computing power, nonlinear estimation of the QUAIDS in itself is not so difficult that linear approximations should be considered. On top of that, the computationally efficient estimation procedure proposed by Blundell and Robin (1999) makes direct estimation of such nonlinear functional forms as the QUAIDS fast enough for practically large models. It does remain the case, however, that linear systems have some advantages over nonlinear systems in empirical applications.

Nonlinearity in parameters becomes problematic, for example, when one tries to take differences to remove nonstationarity in time series. Nonstationary prices and expenditure are often observed in time series, to which flexible demand systems have been applied in many studies. Unless linearly approximated, nonlinear systems including the AIDS and QUAIDS are not amenable to the usual techniques for dealing with nonstationary variables such as cointegration or error correction models. For example, Anderson and Blundell (1983), Burton and Young (1992), Ng (1995), Asche et al. (1997), Attfield (1997), Adam (1999), and Karagiannis et al. (2000) used the linear approximate AIDS in the context of cointegration or error correction models. If the QUAIDS can be as appropriately linearized as the AIDS, therefore, then this theoretically and empirically attractive demand system can be handled more properly in the presence of nonstationarity.

This paper is the first to investigate, in the spirit of linearizing the AIDS, whether adequate linear approximations to the QUAIDS can be obtained. Proposing a new composite variable to approximate a price aggregator function and conducting a simulation, we compare the approximation accuracy of several linearized systems.

QUAIDS and linear approximations

Let $\mathbf{p} = (p_1, \ldots, p_n)$ denote the nominal price vector of n goods and y denote the total expenditure on the goods (expenditure, for short) for each individual. The indirect utility function of the QUAIDS can be specified as

$$[\log V(\mathbf{p}, y)]^{-1} = \left[\frac{\log y - \log f(\mathbf{p})}{g(\mathbf{p})}\right]^{-1} + h(\mathbf{p}), \tag{1}$$

where log is the natural logarithm and $f(\mathbf{p})$, $g(\mathbf{p})$, and $h(\mathbf{p})$ are distinct price aggregator functions defined as

$$\log f(\mathbf{p}) = \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \log p_i \log p_j, \tag{2}$$

$$\log g(\mathbf{p}) = \beta_0 + \sum_i \beta_i \log p_i, \tag{3}$$

$$h(\mathbf{p}) = \eta_0 + \sum_i \eta_i \log p_i. \tag{4}$$

$\sum_i$ is an abbreviated notation for $\sum_{i=1}^n$. $f(\mathbf{p})$ is homogeneous of degree one and $g(\mathbf{p})$ and $h(\mathbf{p})$ are homogeneous of degree zero in $\mathbf{p}$, so that $V(\mathbf{p}, y)$ is homogeneous of degree zero in $\mathbf{p}$ and y, as required. It is assumed, therefore, that the parameters meet the following restrictions,

$$\sum_i \alpha_i = 1, \tag{5}$$

$$\sum_i \beta_i = 0, \tag{6}$$

$$\sum_i \eta_i = 0, \tag{7}$$

$$\sum_i \gamma_{ij} = 0, \qquad j = 1, \ldots, n, \tag{8}$$

$$\sum_j \gamma_{ij} = 0, \qquad i = 1, \ldots, n, \tag{9}$$

which jointly ensure that the resulting demand system fulfills adding-up and homogeneity. Slutsky symmetry is guaranteed by the additional restriction:

$$\gamma_{ij} = \gamma_{ji}, \qquad i, j = 1, \ldots, n. \tag{10}$$

By applying the logarithmic form of Roy's identity $w_i = -(\partial \log V / \partial \log p_i)/(\partial \log V / \partial \log y)$ to equation (1), the QUAIDS is derived as

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \frac{y}{f(\mathbf{p})} + \frac{\eta_i}{g(\mathbf{p})} \left[\log \frac{y}{f(\mathbf{p})} \right]^2, \qquad i = 1, \ldots, n, \tag{11}$$

where w_i denotes the expenditure share (share, for short) of good i for each individual.

The expenditure and uncompensated price elasticities in equation (11) are obtained from

$$\epsilon_i = 1 + \frac{\beta_i}{w_i} + \frac{2\eta_i}{w_i g(\mathbf{p})} \log \frac{y}{f(\mathbf{p})}, \qquad i = 1, \ldots, n, \tag{12}$$

$$\begin{aligned} \epsilon_{ij} = & - \delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i} \left(\alpha_j + \sum_k \gamma_{jk} \log p_k \right) \\ & - \frac{\eta_i}{w_i g(\mathbf{p})} \left[2 \left(\alpha_j + \sum_k \gamma_{jk} \log p_k \right) + \beta_j \log \frac{y}{f(\mathbf{p})} \right] \log \frac{y}{f(\mathbf{p})}, \quad i, j = 1, \ldots, n, \end{aligned} \tag{13}$$

respectively, where δ_{ij} denotes the Kronecker delta, which is equal to unity if $i = j$ and zero otherwise. Being linear in $\log y$, the expenditure elasticities in the QUAIDS can vary with expenditure. Therefore, the QUAIDS can capture such expenditure variations as cannot be captured by the AIDS, where $\eta_i = 0$ for all i and the expenditure elasticities are independent of expenditure.

Linear approximations to the QUAIDS requires that both $f(\mathbf{p})$ and $g(\mathbf{p})$ be replaced with composite variables, which are free of unknown parameters. The most usual composite variable for the approximation to the translog aggregator function $f(\mathbf{p})$ in the AIDS is the Stone price index suggested by Deaton and Muellbauer (1980),

$$\log P^* = \sum_i \bar{w}_i \log p_i, \tag{14}$$

where $\bar{w}_i = E(w_i y)/E(y)$. Expectations denote averaging over all individuals in the economy at a given point of time. Moschini (1995), in the context of the AIDS, showed that employing P^* in place of $f(\mathbf{p})$ can seriously bias elasticity estimates partly because this price index is influenced by changes in units of measurement, and suggested using the following alternative price indices instead,

$$\log P^T = \frac{1}{2} \sum_i \left(\bar{w}_i + \bar{w}_i^0\right) \log \frac{p_i}{p_i^0}, \tag{15}$$

$$\log P^S = \sum_i \bar{w}_i \log \frac{p_i}{p_i^0}, \tag{16}$$

$$\log P^C = \sum_i \bar{w}_i^0 \log p_i, \tag{17}$$

where superscript 0 stands for the base point. P^T is the Törnqvist price index, P^S is the loglinear analogue of the Paasche price index, and P^C is the loglinear analogue of the Laspeyres price index, all of which are exact for a linearly homogeneous Cobb–Douglas aggregator function (Diewert, 1981) and invariant to changes in units.[1] Also exact for a linearly homogeneous translog aggregator function such as $f(\mathbf{p})$, P^T is what Diewert (1976) called a superlative index number. P^S and P^C can be considered two simplified forms of P^T. P^* becomes equivalent to P^S when all prices are scaled to unity at the base point, as is often the case in practice.

In order to approximate $g(\mathbf{p})$, on the other hand, we propose P^Z, a composite variable given by the following equation:

$$\log P^Z = \sum_i (\bar{w}_i - \bar{w}_i^0) \log \frac{p_i}{p_i^0}. \tag{18}$$

[1] Employed in Nicol (1991) and Moosa and Baxter (2002), $\log P^C$ is equivalent to $\sum_i \bar{w}_i^0 \log(p_i/p_i^0)$ because $\sum_i \bar{w}_i^0 \log p_i^0$ is constant and unimportant in estimation.

P^Z may be viewed as a zero degree homogeneous analogue of P^T, and is also invariant to changes in units. Blundell et al. (1993) proposed an extension of the AIDS by simply adding to it quadratic logarithmic expenditure terms. Using unity instead of $g(\mathbf{p})$, their demand system is regarded as an approximate version of the QUAIDS.[2] For comparative purposes, the approximation accuracy of 1, in addition to that of P^Z, is examined in a simulation study in the next section. It should be noted that the linear approximations discussed in the present paper are purely for practical convenience, since all the linearized systems can no longer be derived from the maximization of explicit utility functions without imposing restrictive constraints that remove flexibility.

Equations (12) and (13) may be modified for the elasticities in each linear approximate QUAIDS as

$$\epsilon_i^{LA} = 1 + \frac{\beta_i}{w_i} + \frac{2\eta_i}{w_i P^g} \log \frac{y}{P^f}, \qquad i = 1, \ldots, n, \tag{19}$$

$$\epsilon_{ij}^{LA} = -\,\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i} \frac{\partial \log P^f}{\partial \log p_j} - \frac{\eta_i}{w_i P^g} \left[2 \frac{\partial \log P^f}{\partial \log p_j} + \left(\frac{\partial \log P^g}{\partial \log p_j} \right) \log \frac{y}{P^f} \right] \log \frac{y}{P^f}, \quad i, j = 1, \ldots, n, \tag{20}$$

respectively, where

$$\frac{\partial \log P^f}{\partial \log p_j} \begin{cases} \approx \bar{w}_j, & \text{if } P^f = P^* \text{ or } P^f = P^S, \\ \approx \dfrac{\bar{w}_j + \bar{w}_j^0}{2}, & \text{if } P^f = P^T, \\ = \bar{w}_j^0, & \text{if } P^f = P^C, \end{cases} \tag{21}$$

$$\frac{\partial \log P^g}{\partial \log p_j} \begin{cases} \approx \bar{w}_j - \bar{w}_j^0, & \text{if } P^g = P^Z, \\ = 0, & \text{if } P^g = 1. \end{cases} \tag{22}$$

Because $\partial \bar{w}_i / \partial \log p_j \neq 0$, equation (21) is not an exact equality but only an approximate expression when P^f equals P^*, P^S, or P^T, and equation (22) is also an approximation when $P^g = P^Z$. In the case of P^* or P^S, however, equation (21) has commonly been used to calculate the price elasticities in the linearized AIDS (e.g.,

[2] If the model of Blundell et al. (1993) is to be built on utility maximization, then the ratio of η_i to β_i is forced to be the same for all i, that is, the total number of free parameters is reduced to that of the AIDS. If η_i and β_i are to be independent of each other for all i, therefore, then their model cannot be derived from utility maximization.

Goddard, 1983; Chalfant, 1987; Moschini and Meilke, 1989), and its approximation accuracy was illustrated by Green and Alston (1990), Alston et al. (1994), and Buse (1994). Equation (21) in the case of P^T and equation (22) in the case of P^Z are based on this analogy. In general, the elasticities calculated by the linear approximate formulas are different from those calculated by the original nonlinear formulas. Equations (19) and (20) become the same as equations (12) and (13), respectively, only if α_0 is set to zero, expenditure and all prices are scaled to unity at the base point, and then the elasticities are evaluated at the base point (note that $\alpha_i = w_i^0$).[3]

A simulation

In order to compare the performance of the linearized and nonlinear systems, a Monte Carlo experiment is conducted based on actual Japanese annual data on prices and per capita expenditure for the period 1965 through 1999 on five categories of food: (1) nonglutinous rice (rice), (2) fresh fish and shellfish (fish), (3) fresh meat (meat), (4) fresh vegetables (vegetables), and (5) fresh fruits (fruits). These data are obtained from *Annual Report on the Family Income and Expenditure Survey* published by the Statistics Bureau of the Ministry of Public Management, Home Affairs, Posts, and Telecommunications.

Part of true elasticities at the sample mean and true parameters are preselected so that $\epsilon_1 = -0.20$, $\epsilon_2 = 1.30$, $\epsilon_3 = 1.50$, $\epsilon_4 = 0.80$, $\epsilon_{11} = -0.30$, $\epsilon_{12} = 0.15$, $\epsilon_{13} = 0.15$, $\epsilon_{14} = 0.10$, $\epsilon_{22} = -0.80$, $\epsilon_{23} = -0.15$, $\epsilon_{24} = -0.15$, $\epsilon_{33} = -0.90$, $\epsilon_{34} = -0.15$, $\epsilon_{44} = -0.60$, $\eta_1 = 0.004$, $\eta_2 = -0.001$, $\eta_3 = -0.001$, and $\eta_4 = -0.001$.[4] The other true elasticities can be calculated through the restrictions of aggregation ($\sum_i w_i\epsilon_i = 1$ or $w_j + \sum_i w_i\epsilon_{ij} = 0$ for all j), homogeneity ($\epsilon_i + \sum_j \epsilon_{ij} = 0$ for all i) and Slutsky symmetry [$(\epsilon_{ij} + w_j\epsilon_i)w_i = (\epsilon_{ji} + w_i\epsilon_j)w_j$ for all i and j]. The remaining 18 true free parameters are calibrated by solving a set of 14 elasticity equations and four share equations. Given these calibrated α_i, β_i and γ_{ij} and preselected η_i, 1000 samples, each of which consists of 35 observations of four shares, are generated by

[3] Estimation results of the nonlinear QUAIDS given by equations (12) are found to be virtually unchanged whether α_0 is estimated or fixed at any particular value including zero.

[4] With parameter restrictions (5)–(10) imposed and the values of inconsequential parameters α_0 and β_0 preset to zero, the total number of free parameters of the QUAIDS is $(n+6)(n-1)/2$, which is $n-1$ more than the number required for parsimonious flexible functional forms (Pollak and Wales, 1992) such as the AIDS. The additional $n-1$ parameters are necessary for quadratic logarithmic expenditure terms.

adding to the right-hand side of equation (11) random draws from a multivariate normal distribution with mean zero and the following covariance matrix:[5]

$$\Omega = 10^{-5} \begin{pmatrix} 0.50 & -0.15 & -0.15 & -0.10 \\ -0.15 & 0.45 & -0.10 & -0.05 \\ -0.15 & -0.10 & 0.45 & -0.05 \\ -0.10 & -0.05 & -0.05 & 0.40 \end{pmatrix}. \quad (23)$$

For each of the 1000 replicated matrices of shares of dimension 35×4, the nonlinear system and the linearized systems are estimated using iterative seemingly unrelated regression to calculate and compare elasticities.[6] The base point is set to the sample mean, where, for the sake of generality, expenditure and all prices are not scaled to unity.[7]

The averages and standard deviations of 1000 estimates are presented for the expenditure and uncompensated own price elasticities evaluated at the sample mean (tables 1 and 2), first sample point (tables 3 and 4), and last sample point (tables 5 and 6).[8] Overall, the three systems linearized with P^Z and the system linearized with $(P^C, 1)$ give results relatively similar to those of the nonlinear system when the corresponding linear approximate formulas are used, although the approximation accuracy decreases in the uncompensated own price elasticities evaluated at the last sample point. When the corresponding approximate formulas are used, the systems linearized with $(P^T, 1)$ and $(P^S, 1)$ perform as well at the sample mean and first sample point as, but worse at the last sample point, especially for the expenditure elasticities, than the systems linearized with (P^T, P^Z), (P^S, P^Z), (P^C, P^Z), and $(P^C, 1)$. Using the original nonlinear formulas for the linearized systems produces poor results, except for the expenditure elasticities in the case of (P^C, P^Z). In particular, the magnitude of the elasticities are extremely unreasonable when the original nonlinear formulas are used for the systems linearized with 1. It follows that, in general, the elasticities in the linearized systems should be computed using the corresponding approximate formulas.

[5] A covariance matrix for five goods, Ω is given based on the analogy of the covariance matrix used by Moschini (1995), who conducted an investigation into linear approximations to the AIDS in a four-good case.

[6] The simulation is performed using SHAZAM 10.0.

[7] Otherwise there is no difference between the linear approximate and original nonlinear formulas at the sample mean.

[8] As expected, using P^* yields unambiguously poor results, as in the case of the AIDS shown by Moschini (1995).

Conclusions

This paper has proposed a new composite variable to linearize the recently popular QUAIDS. Comparing the estimated elasticities in the several linearized systems and the original nonlinear system, the simulation study based on Japanese demand for food including rice has drawn the following conclusions.[9] First, the newly introduced composite variable performs best in combination with the price indices that were shown by Moschini (1995) to be appropriate for linearizing the AIDS. The approximation accuracy can decrease, however, when the point of evaluation of elasticities is away from the base point of the composite variables. Secondly, Blundell et al. (1993) type of approximation produces as good results when combined with the loglinear Laspeyres price index. However, using their type of approximation along with the Törnqvist price index or the loglinear Paasche price index can perform worse than using the new composite variable when the point of evaluation is away from the base point. Finally, but not least, the corresponding approximate formulas instead of the original nonlinear formulas should be used to calculate the elasticities in the linearized systems. Both formulas yield similar results only for the expenditure elasticities in the system linearized with the new composite variable and the loglinear Laspeyres price index.

These results suggest that the QUAIDS can be linearly approximated on a practical basis if the appropriate combinations of composite variables and elasticity formulas are employed and the base point is set to the point of evaluation. Giving the QUAIDS a chance to have a broader range of application, the linear approximations discussed in the present paper are especially useful when one uses nonstationary time series, to which nonlinear systems are difficult to apply properly.

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[9] Our simulation exercise is almost as limited in scope and depth as the simulation study of the AIDS by Moschini (1995). Buse and Chan (2000) further investigated the linearized AIDS by performing a more comprehensive simulation. Applying their type of Monte Carlo design to the QUAIDS is a possible direction for future research.

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Table 1. Averages and standard deviations of estimated expenditure elasticities at the sample mean

	Rice	Fish	Meat	Vegetables	Fruits
True value	-0.20	1.30	1.50	0.80	1.94
Nonlinear system	-0.20 (0.07)	1.29 (0.08)	1.52 (0.13)	0.80 (0.10)	1.94 (0.18)
System linearized with		Using corresponding linear approximate formula			
(P^T, P^Z)	-0.21 (0.07)	1.30 (0.09)	1.51 (0.14)	0.79 (0.10)	1.97 (0.18)
(P^S, P^Z)	-0.21 (0.07)	1.30 (0.09)	1.51 (0.14)	0.79 (0.10)	1.97 (0.18)
(P^C, P^Z)	-0.21 (0.07)	1.30 (0.09)	1.51 (0.14)	0.79 (0.10)	1.97 (0.18)
$(P^T, 1)$	-0.21 (0.08)	1.30 (0.08)	1.49 (0.14)	0.80 (0.10)	1.96 (0.19)
$(P^S, 1)$	-0.19 (0.08)	1.30 (0.08)	1.48 (0.13)	0.81 (0.10)	1.95 (0.19)
$(P^C, 1)$	-0.22 (0.08)	1.30 (0.08)	1.51 (0.14)	0.79 (0.10)	1.97 (0.19)
		Using original nonlinear formula			
(P^T, P^Z)	-0.30 (0.12)	1.32 (0.15)	1.54 (0.20)	0.77 (0.14)	2.05 (0.27)
(P^S, P^Z)	-0.38 (0.14)	1.34 (0.16)	1.57 (0.21)	0.75 (0.15)	2.12 (0.29)
(P^C, P^Z)	-0.21 (0.08)	1.30 (0.10)	1.51 (0.15)	0.79 (0.10)	1.97 (0.19)
$(P^T, 1)$	-1.49E+10 (3.18E+11)	-1.48E+10 (3.39E+11)	-8.16E+09 (2.41E+11)	3.29E+10 (7.65E+11)	9.10E+09 (1.88E+11)
$(P^S, 1)$	-1.27E+08 (3.13E+09)	-3.05E+08 (9.33E+09)	-2.36E+08 (7.03E+09)	5.72E+08 (1.72E+10)	1.69E+08 (4.08E+09)
$(P^C, 1)$	-1.95E+07 (4.76E+08)	-8.05E+06 (2.02E+08)	-1.29E+06 (5.83E+07)	2.42E+07 (5.76E+08)	8.68E+06 (1.63E+08)

Note: Numbers other than true values are averages of 1000 estimates and, on the right side of them in parentheses, the corresponding standard deviations. Each elasticity is evaluated at the sample mean.

Table 2. Averages and standard deviations of estimated uncompensated own price elasticities at the sample mean

	Rice	Fish	Meat	Vegetables	Fruits
True value	-0.30	-0.80	-0.90	-0.60	-1.01
Nonlinear system	-0.30 (0.03)	-0.80 (0.03)	-0.91 (0.06)	-0.60 (0.03)	-1.02 (0.04)
System linearized with		Using corresponding linear approximate formula			
(P^T, P^Z)	-0.30 (0.03)	-0.79 (0.04)	-0.91 (0.07)	-0.60 (0.03)	-1.03 (0.04)
(P^S, P^Z)	-0.30 (0.03)	-0.79 (0.04)	-0.91 (0.07)	-0.60 (0.03)	-1.03 (0.04)
(P^C, P^Z)	-0.30 (0.03)	-0.79 (0.04)	-0.91 (0.07)	-0.60 (0.03)	-1.03 (0.04)
$(P^T, 1)$	-0.31 (0.03)	-0.81 (0.03)	-0.90 (0.06)	-0.61 (0.03)	-1.02 (0.04)
$(P^S, 1)$	-0.32 (0.03)	-0.81 (0.03)	-0.89 (0.06)	-0.61 (0.03)	-1.01 (0.04)
$(P^C, 1)$	-0.30 (0.03)	-0.80 (0.03)	-0.91 (0.07)	-0.60 (0.03)	-1.03 (0.04)
		Using original nonlinear formula			
(P^T, P^Z)	3.82 (0.75)	-0.51 (0.21)	-0.14 (0.43)	-0.43 (0.18)	0.78 (0.84)
(P^S, P^Z)	4.30 (0.84)	-0.48 (0.23)	-0.06 (0.47)	-0.41 (0.20)	0.98 (0.93)
(P^C, P^Z)	1.92 (0.43)	-0.62 (0.14)	-0.47 (0.24)	-0.50 (0.12)	-0.03 (0.48)
$(P^T, 1)$	5.46E+12 (1.16E+14)	4.94E+12 (1.09E+14)	2.08E+12 (6.25E+13)	2.33E+13 (5.55E+14)	1.49E+12 (2.65E+13)
$(P^S, 1)$	2.20E+10 (4.62E+11)	1.07E+11 (3.09E+12)	5.72E+10 (1.73E+12)	3.42E+11 (1.04E+13)	2.76E+10 (5.81E+11)
$(P^C, 1)$	3.28E+09 (8.68E+10)	5.75E+08 (1.35E+10)	1.63E+08 (2.58E+09)	4.49E+09 (1.11E+11)	4.98E+08 (7.36E+09)

Note: See notes to table 1.

Table 3. Averages and standard deviations of estimated expenditure elasticities at the first sample point

	Rice	Fish	Meat	Vegetables	Fruits
Nonlinear system	-0.20 (0.08)	1.30 (0.10)	1.47 (0.13)	0.78 (0.11)	1.88 (0.17)
System linearized with			Using corresponding linear approximate formula		
(P^T, P^Z)	-0.22 (0.07)	1.33 (0.10)	1.45 (0.13)	0.77 (0.11)	1.90 (0.17)
(P^S, P^Z)	-0.23 (0.07)	1.33 (0.10)	1.45 (0.13)	0.77 (0.11)	1.90 (0.17)
(P^C, P^Z)	-0.22 (0.07)	1.33 (0.10)	1.45 (0.13)	0.77 (0.11)	1.90 (0.17)
$(P^T, 1)$	-0.24 (0.11)	1.34 (0.12)	1.45 (0.15)	0.78 (0.14)	1.92 (0.22)
$(P^S, 1)$	-0.24 (0.10)	1.35 (0.11)	1.43 (0.14)	0.79 (0.13)	1.91 (0.21)
$(P^C, 1)$	-0.23 (0.13)	1.31 (0.13)	1.46 (0.16)	0.77 (0.15)	1.92 (0.23)
			Using original nonlinear formula		
(P^T, P^Z)	-0.30 (0.12)	1.35 (0.16)	1.48 (0.17)	0.75 (0.15)	1.97 (0.24)
(P^S, P^Z)	-0.38 (0.14)	1.37 (0.17)	1.51 (0.18)	0.73 (0.16)	2.03 (0.26)
(P^C, P^Z)	-0.22 (0.08)	1.33 (0.11)	1.45 (0.13)	0.77 (0.11)	1.90 (0.17)
$(P^T, 1)$	-4.69E+10 (1.18E+12)	-5.34E+10 (1.52E+12)	-3.42E+10 (1.01E+12)	1.25E+11 (3.49E+12)	2.97E+10 (7.25E+11)
$(P^S, 1)$	-4.07E+08 (1.04E+10)	-1.01E+09 (3.31E+10)	-6.94E+08 (2.04E+10)	1.92E+09 (6.00E+10)	5.04E+08 (1.24E+10)
$(P^C, 1)$	-3.48E+07 (7.36E+08)	-1.45E+07 (3.53E+08)	-5.53E+06 (1.45E+08)	4.83E+07 (1.02E+09)	1.55E+07 (2.52E+08)

Note: Numbers are averages of 1000 estimates and, on the right side of them in parentheses, the corresponding standard deviations. Each elasticity is evaluated at the first sample point.

Table 4. Averages and standard deviations of estimated uncompensated own price elasticities at the first sample point

	Rice	Fish	Meat	Vegetables	Fruits
Nonlinear system	-0.25 (0.09)	-0.76 (0.04)	-0.91 (0.06)	-0.56 (0.05)	-0.98 (0.06)
System linearized with		Using corresponding linear approximate formula			
(P^T, P^Z)	-0.29 (0.03)	-0.77 (0.04)	-0.91 (0.05)	-0.57 (0.04)	-1.03 (0.03)
(P^S, P^Z)	-0.29 (0.03)	-0.77 (0.04)	-0.91 (0.05)	-0.57 (0.04)	-1.03 (0.03)
(P^C, P^Z)	-0.29 (0.03)	-0.77 (0.04)	-0.91 (0.05)	-0.57 (0.04)	-1.03 (0.03)
$(P^T, 1)$	-0.30 (0.03)	-0.79 (0.04)	-0.91 (0.06)	-0.57 (0.04)	-1.03 (0.04)
$(P^S, 1)$	-0.31 (0.03)	-0.79 (0.04)	-0.91 (0.06)	-0.58 (0.04)	-1.03 (0.03)
$(P^C, 1)$	-0.30 (0.03)	-0.77 (0.04)	-0.92 (0.06)	-0.57 (0.04)	-1.03 (0.04)
		Using original nonlinear formula			
(P^T, P^Z)	3.86 (0.75)	-0.45 (0.24)	-0.24 (0.38)	-0.38 (0.19)	0.65 (0.77)
(P^S, P^Z)	4.32 (0.83)	-0.42 (0.26)	-0.18 (0.42)	-0.36 (0.21)	0.83 (0.86)
(P^C, P^Z)	1.95 (0.43)	-0.58 (0.16)	-0.53 (0.22)	-0.46 (0.13)	-0.10 (0.44)
$(P^T, 1)$	1.63E+13 (3.80E+14)	1.89E+13 (5.12E+14)	9.47E+12 (2.78E+14)	9.58E+13 (2.74E+15)	5.48E+12 (1.13E+14)
$(P^S, 1)$	7.68E+10 (1.69E+12)	3.81E+11 (1.15E+13)	1.77E+11 (5.26E+12)	1.22E+12 (3.82E+13)	9.44E+10 (2.08E+12)
$(P^C, 1)$	5.68E+09 (1.32E+11)	1.13E+09 (2.36E+10)	5.44E+08 (9.64E+09)	9.14E+09 (2.00E+11)	9.79E+08 (1.30E+10)

Note: See notes to table 3.

Table 5. Averages and standard deviations of estimated expenditure elasticities at the last sample point

	Rice	Fish	Meat	Vegetables	Fruits
Nonlinear system	-0.03 (0.06)	1.30 (0.08)	1.59 (0.15)	0.81 (0.09)	2.08 (0.20)
System linearized with			Using corresponding linear approximate formula		
(P^T, P^Z)	-0.03 (0.06)	1.31 (0.10)	1.58 (0.16)	0.80 (0.10)	2.11 (0.21)
(P^S, P^Z)	-0.04 (0.06)	1.31 (0.10)	1.58 (0.16)	0.80 (0.10)	2.11 (0.21)
(P^C, P^Z)	-0.03 (0.06)	1.31 (0.10)	1.58 (0.16)	0.80 (0.10)	2.11 (0.21)
$(P^T, 1)$	0.05 (0.13)	1.28 (0.15)	1.54 (0.18)	0.82 (0.15)	1.99 (0.30)
$(P^S, 1)$	0.13 (0.13)	1.23 (0.15)	1.53 (0.17)	0.83 (0.15)	1.93 (0.30)
$(P^C, 1)$	-0.03 (0.13)	1.34 (0.14)	1.56 (0.18)	0.82 (0.15)	2.06 (0.30)
			Using original nonlinear formula		
(P^T, P^Z)	-0.12 (0.12)	1.33 (0.17)	1.63 (0.24)	0.77 (0.15)	2.21 (0.32)
(P^S, P^Z)	-0.20 (0.13)	1.36 (0.18)	1.67 (0.25)	0.76 (0.15)	2.30 (0.34)
(P^C, P^Z)	-0.03 (0.07)	1.31 (0.11)	1.58 (0.17)	0.80 (0.10)	2.11 (0.23)
$(P^T, 1)$	-5.28E+09 (1.27E+11)	-5.29E+09 (1.33E+11)	-1.55E+09 (4.08E+10)	1.05E+10 (2.49E+11)	3.90E+09 (7.80E+10)
$(P^S, 1)$	-2.44E+07 (4.12E+08)	-3.77E+07 (1.07E+09)	-3.47E+07 (8.96E+08)	6.98E+07 (1.81E+09)	4.58E+07 (8.55E+08)
$(P^C, 1)$	-1.68E+07 (4.47E+08)	-7.88E+06 (2.29E+08)	4.88E+05 (5.02E+07)	2.23E+07 (6.07E+08)	8.96E+06 (2.03E+08)

Note: Numbers are averages of 1000 estimates and, on the right side of them in parentheses, the corresponding standard deviations. Each elasticity is evaluated at the last sample point.

Table 6. Averages and standard deviations of estimated uncompensated own price elasticities at the last sample point

	Rice	Fish	Meat	Vegetables	Fruits
Nonlinear system	-0.51 (0.06)	-0.84 (0.06)	-0.92 (0.08)	-0.66 (0.05)	-1.12 (0.09)
System linearized with	Using corresponding linear approximate formula				
(P^T, P^Z)	-0.40 (0.02)	-0.79 (0.04)	-0.89 (0.09)	-0.62 (0.03)	-1.03 (0.05)
(P^S, P^Z)	-0.40 (0.02)	-0.79 (0.04)	-0.89 (0.09)	-0.62 (0.03)	-1.03 (0.05)
(P^C, P^Z)	-0.41 (0.02)	-0.79 (0.04)	-0.89 (0.09)	-0.62 (0.03)	-1.03 (0.05)
$(P^T, 1)$	-0.41 (0.04)	-0.79 (0.04)	-0.87 (0.06)	-0.63 (0.03)	-1.00 (0.06)
$(P^S, 1)$	-0.42 (0.05)	-0.79 (0.04)	-0.85 (0.06)	-0.63 (0.03)	-0.98 (0.06)
$(P^C, 1)$	-0.41 (0.04)	-0.80 (0.04)	-0.89 (0.07)	-0.63 (0.03)	-1.02 (0.05)
	Using original nonlinear formula				
(P^T, P^Z)	3.15 (0.67)	-0.49 (0.22)	0.01 (0.50)	-0.45 (0.18)	1.06 (0.98)
(P^S, P^Z)	3.59 (0.76)	-0.46 (0.25)	0.10 (0.56)	-0.43 (0.20)	1.30 (1.10)
(P^C, P^Z)	1.49 (0.37)	-0.61 (0.14)	-0.38 (0.27)	-0.52 (0.11)	0.11 (0.54)
$(P^T, 1)$	2.15E+12 (5.42E+13)	1.80E+12 (4.29E+13)	3.56E+11 (9.78E+12)	6.81E+12 (1.61E+14)	6.99E+11 (1.24E+13)
$(P^S, 1)$	5.65E+09 (1.11E+11)	1.52E+10 (3.19E+11)	7.89E+09 (1.98E+11)	3.67E+10 (9.84E+11)	1.15E+10 (2.88E+11)
$(P^C, 1)$	2.98E+09 (8.32E+10)	5.77E+08 (1.56E+10)	9.30E+07 (1.36E+09)	4.20E+09 (1.18E+11)	4.70E+08 (8.80E+09)

Note: See notes to table 5.